

C1 June 2018 (MA)

$$\text{Q1i)} \quad \frac{\sqrt{48}}{3} - \frac{6}{\sqrt{3}} = \frac{\sqrt{16 \times 3}}{3} - \frac{6\sqrt{3}}{3} = \frac{4\sqrt{3}}{3} - 2\sqrt{3} = \boxed{2\sqrt{3}}$$

$$\text{ii)} \quad 3^{6x-3} = 81$$

$$3^{6x-3} = 3^4$$

$$\therefore 6x - 3 = 4$$

$$6x = 7$$

$$\boxed{x = \frac{7}{6}}$$

$$\text{Q2a)} \quad \int [3x^{\frac{1}{2}} - 6x + 4] dx = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{6x^2}{2} + 4x + c$$

$$= \boxed{2x^{3/2} - 3x^2 + 4x + c}$$

$$\text{bi)} \quad y = 3x^{\frac{1}{2}} - 6x + 4$$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 6$$

$$\text{ii)} \quad \frac{3}{2}x^{-\frac{1}{2}} - 6 = 0$$

$$\frac{3}{2\sqrt{x}} = 6 \quad \therefore \frac{2\sqrt{x}}{3} = \frac{1}{6}$$

$$\text{So } \sqrt{x} = \frac{3}{2} \times \frac{1}{6} = \frac{3}{12} = \frac{1}{4} //$$

$$\sqrt{x} = \frac{1}{4}$$

$$\boxed{x = \frac{1}{16}}$$

$$(Q3a) \quad f(x) = x^2 - 10x + 23$$

$$= (x-5)^2 - 25 + 23$$

$$= \boxed{(x-5)^2 - 2} \quad \begin{array}{l} a = -5 \\ b = -2 \end{array}$$

$$b) \quad (x-5)^2 - 2 = 0$$

$$(x-5)^2 = 2$$

$$x-5 = \pm\sqrt{2}$$

$$x = 5 \pm \sqrt{2} \quad \text{so}$$

$$\boxed{\begin{array}{l} x = 5 + \sqrt{2} \\ x = 5 - \sqrt{2} \end{array}}$$

$$c) \quad \text{let } x = y^{\frac{1}{2}},$$

then the eqn in (c) is identical to (b).

$$\text{so } \dots \quad y^{\frac{1}{2}} = 5 + \sqrt{2}$$

$$y = (5 + \sqrt{2})^2 = 25 + 10\sqrt{2} + 2$$

$$= \boxed{27 + 10\sqrt{2}} //$$

Q4a)

year 1	year 2	year 3	}	$a = 600$
600	720	840		$d = 120$
$a$	$a + d$	$a + 2d$		

$$10^{\text{th}} \text{ term} = a + (10-1)d = 600 + 120(9) = \boxed{\pounds 1680}$$

b)

year 1	year 2	year 3	}	$a = 130$
130	210	290		$d = 80$
$a$	$a + d$	$a + 2d$		

For Kim:  $S_N = \frac{N}{2} [2a + (n-1)d]$

$$= \frac{N}{2} [260 + (N-1)80]$$

For Andy:  $S_N = \frac{N}{2} [1200 + (N-1)120]$

we are told...  $\frac{N}{2} [1200 + (N-1)120] = 2 \times \frac{N}{2} [260 + (N-1)80]$

$$\Rightarrow 1200 + 120N - 120 = 520 + 160N - 160$$

$$\Rightarrow 40N = 720$$

$$\therefore \boxed{N = 18}$$

Q5a)  $+2$  in the  $x$ -direction  $\rightarrow$   $(4, 7)$

b)  $\times \frac{1}{2}$  to all  $x$ -values  $\rightarrow$   $(\frac{5}{2}, 6)$

c)  $y = 1$  (only  $x$ -values change).

d)  $y = 7$  (at highest point).

and  $y \leq 1$

Q6a)  $a_2 = \frac{4}{4+1} = \frac{4}{5}$

$a_3 = \frac{\frac{4}{5}}{\frac{4}{5}+1} = \frac{\frac{4}{5}}{\frac{9}{5}} = \frac{4}{9}$

$a_4 = \frac{\frac{4}{9}}{\frac{4}{9}+1} = \frac{\frac{4}{9}}{\frac{13}{9}} = \frac{4}{13}$

b) 
$$\begin{array}{r} 2p + q = 5 \rightarrow (a_2) \\ - [3p + q = 9] \rightarrow (a_3) \\ \hline \end{array}$$

$-p + 0 = -4$

$p = 4$  so  $q = 5 - 2(4) = -3$

$$d) a_N = \frac{4}{4N-3} = \frac{4}{321} //$$

$$\therefore 4N-3 = 321$$

$$4N = 324$$

$$N = \frac{324}{4} = \frac{162}{2} = \boxed{81}$$

$$Q7a) 20x^2 = 4kx - 13kx^2 + 2$$

$$20x^2 + 13kx^2 - 4kx - 2 = 0$$

$$(20 + 13k)x^2 + (-4k)x + (-2) = 0$$

No real roots  $\rightarrow b^2 - 4ac < 0$

$$\left. \begin{array}{l} a = 20 + 13k \\ b = -4k \\ c = -2 \end{array} \right\} \begin{array}{l} b^2 - 4ac < 0 \\ (-4k)^2 - 4(20 + 13k)(-2) < 0 \end{array}$$

$$16k^2 + 8(20 + 13k) < 0$$

$$16k^2 + 160 + 104k < 0$$

$$\div 8: 2k^2 + 20 + 13k < 0$$

$$2k^2 + 13k + 20 < 0$$

$$b) \quad 2u^2 + 13u + 20 = 0$$

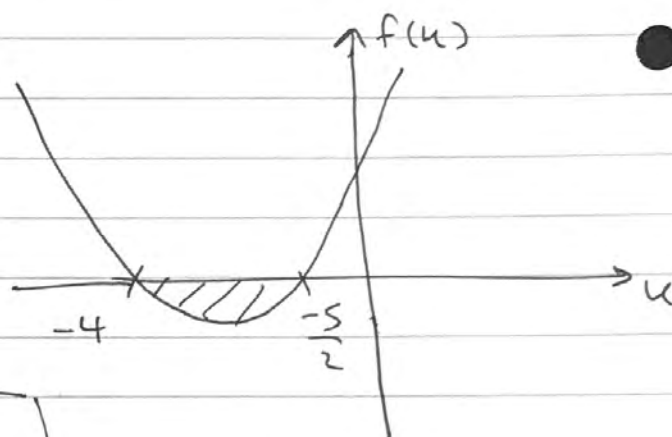
$$\left. \begin{array}{l} a=2 \\ b=13 \\ c=20 \end{array} \right\} u = \frac{-13 \pm \sqrt{169 - 4(2)(20)}}{4} = \frac{-13 \pm \sqrt{9}}{4}$$

$$u = \frac{-13 \pm 3}{4} \quad \text{so} \quad u = \frac{-5}{2}$$

$$u = -4$$

we want where  
 $2u^2 + 13u + 20 < 0$

ie where y-values  $< 0$



$$\text{so } \boxed{-4 < u < -\frac{5}{2}}$$

$$Q8a) \quad \div 4: \quad y = \left(\frac{5}{4}\right)x + 3$$

l, gradient =  $\boxed{\frac{5}{4}}$

$$b) \quad y - 5 = \frac{5}{4}(x - 12)$$

$$y = \frac{5}{4}x - 15 + 5$$

$$\boxed{y = \frac{5}{4}x - 10}$$

(i)  $x=0 : y = -10$

(ii)

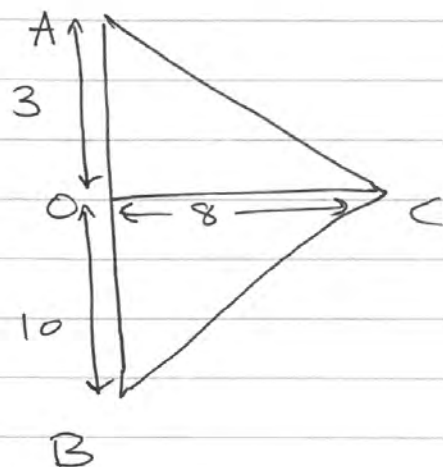
$$y=0 : \frac{5}{4}x = 10 \quad \therefore x = \frac{10 \times 4}{5} = \frac{40}{5} = 8$$

so  $B(0, -10)$        $C(8, 0)$

d) Consider  $\triangle ABC$ .

$$\text{Area } OAC = \frac{1}{2}(8)(3) = 12$$

$$\text{Area } OBC = \frac{1}{2}(10)(8) = 40$$



so Area  $\triangle ABC = 52$

$$\therefore \text{Area parallelogram} = 2 \times 52$$

$$= \boxed{104 \text{ units}^2}$$

(Q9a)  $f'(x) = (x-3)(3x+5) = 3x^2 + 5x - 9x - 15$   
 $= 3x^2 - 4x - 15$

$$\therefore f(x) = \int f'(x) dx = \int (3x^2 - 4x - 15) dx$$

$$= \frac{3x^3}{3} - \frac{4x^2}{2} - 15x + c$$

$$= x^3 - 2x^2 - 15x + c = f(x)$$

$$\text{at } x=1, y=20 : 20 = 1^3 - 2(1)^2 - 15(1) + c$$

$$20 + 16 = c = 36$$

$$\text{so } f(x) = x^3 - 2x^2 - 15x + 36$$

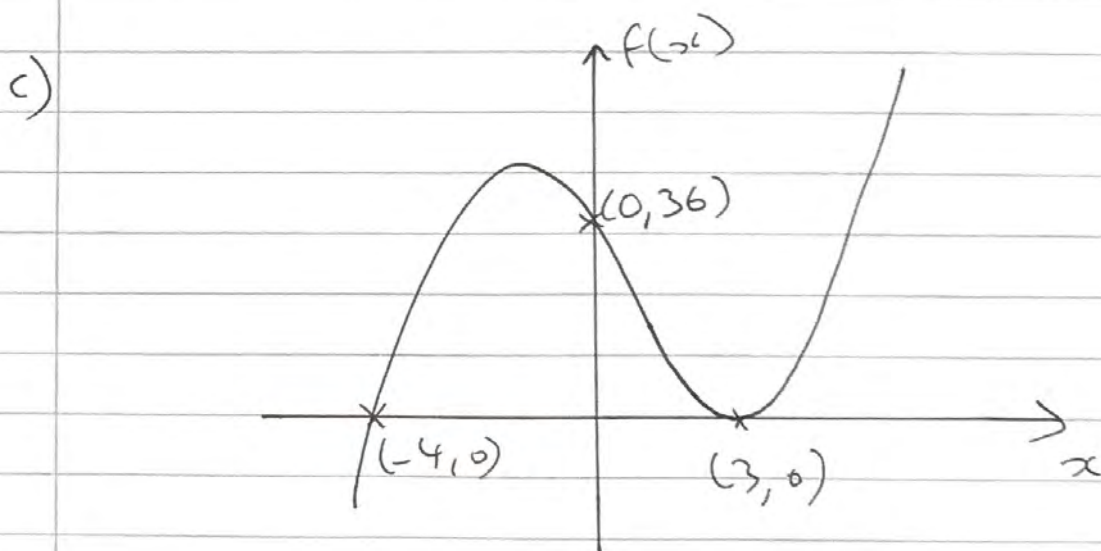
$$b) (x-3)^2(x+A) \rightarrow (x-3)^2(x+4)$$

$$A=4, (36 = 3^2 \times A)$$

$$(x-3)^2(x+4) = (x^2 - 6x + 9)(x+4)$$

$$= (x^3 + 4x^2 - 6x^2 - 24x + 9x + 36)$$

$$= (x^3 - 2x^2 - 15x + 36) = f(x)$$





$$\bullet \text{ (10a)} \quad y = \frac{1}{2}x + 27x^{-1} - 12$$

$$\frac{dy}{dx} = \frac{1}{2} - 27x^{-2}$$

$$\text{at A, } \frac{dy}{dx} = \frac{1}{2} - 27(3^{-2}) = \frac{1}{2} - \frac{27}{9}$$

$$= \frac{1}{2} - 3 = -\frac{5}{2}$$

$$\bullet \text{ So at normal, } m = \frac{2}{5} \quad \left( \frac{2}{5}x - \frac{5}{2} = -1 \right)$$

$$\therefore y - \frac{3}{2} = \frac{2}{5}(x - 3)$$

$$y + \frac{3}{2} = \frac{2}{5}x - \frac{6}{5}$$

$$y = \frac{2}{5}x - \frac{6}{5} - \frac{3}{2}$$

$$y = \frac{2}{5}x - \frac{27}{10}$$

$$\times 10 : \quad \boxed{10y = 4x - 27}$$

$$\bullet \text{ b) } y = \frac{4x}{10} - \frac{27}{10} = \frac{1}{2}x + \frac{27}{x} - 12$$

$$\bullet \text{ } \times 10x : 4x^2 - 27x = 5x^2 + 270 - 120x$$

$$x^2 - 93x + 270 = 0 //$$

$$(x-90)(x-3) = 0$$

$$x = 90 //$$

$$x = 3 \rightarrow \text{this is A}$$

$$y = \frac{4(90)}{10} - \frac{27}{10} = 33.3$$

$$\text{so } \boxed{B(90, 33.3)}$$